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Self-accelerating solutions of scalar–tensor gravity

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Abstract: Scalar–tensor gravity is the simplest and best understood modification of general relativity, consisting of a real scalar field coupled directly to the Ricci scalar curvature. Models of this type have self-accelerating solutions. In an example inspired by string dilaton couplings, scalar–tensor gravity coupled to ordinary matter exhibits a de Sitter type expansion, even in the presence of a *negative* cosmological constant whose magnitude exceeds that of the matter density. This unusual behavior does not require phantoms, ghosts or other exotic sources. More generally, we show that any expansion history can be interpreted as arising partly or entirely from scalar–tensor gravity. To distinguish any quintessence or inflation model from its scalar–tensor variants, we use the fact that scalar–tensor models imply deviations of the post-Newtonian parameters of general relativity, and time variation of the Newton's gravitational coupling G. We emphasize that next-generation probes of modified GR and the time variation of G are an essential complement to dark energy probes based on luminosity-distance measurements.

Keywords: cosmology, quintessence, dark energy, inflation.

Contents

1. Introduction

There are three phenomenological approaches to explaining the accelerating expansion of the universe. The first is a positive cosmological constant whose magnitude (considered as vacuum energy) somewhat exceeds the current matter density [1]. The second is to introduce dynamical dark energy, usually in the form of an ultralight quintessence scalar field [2]-[5] The third is to modify gravity so as to produce self-accelerating solutions [6]-[12]; usually such solutions can be regarded as modifying the left-hand side of the Einstein equations of motion in such a way as to simulate a dark energy source on the right-hand side of these equations.

Scalar–tensor models [13]-[16] can be regarded as a combination of the latter two approaches. General relativity is modified by a real scalar field $\theta(x)$ that couples directly to the Ricci scalar curvature R. If the vacuum expectation value of θ is dynamically evolving today, then the Einstein equations are modified in a nonlinear way and exhibit new types of solutions. In the absence of sources scalar–tensor models are classically equivalent to higher derivative modified gravity models based on a nonlinear function $f(R)$, but this equivalence almost certainly does not hold in realistic contexts. In fact scalar–tensor models have a big advantage over other approaches to modified gravity, in that it is transparent to identify regimes in these models that are weakly–coupled and free of ghosts, violations of the dominant energy condition, and other pathologies.

The existence of self-accelerating solutions of scalar–tensor gravity models has already received considerable attention [17]-[21]. Strong observational constraints on such scalar–tensor cosmologies have been derived and discussed in the literature [22]-[26]. Our purpose here is to exhibit some simple analytic solutions that demonstrate the promise, weaknesses and generality of accelerated expansion from scalar–tensor gravity. Although we will focus on the connection to dark energy, most of our analysis is also relevant for building models of primordial inflation.

In all of our examples the real scalar is ultralight. There are two known motivations for such fields. The first is string theory, in which the low energy effective action can exhibit a massless dilaton and other massless moduli fields. These generally have exponential couplings to the Ricci scalar. They may or may not have direct couplings to matter; when they do have such couplings, these may be sufficiently universal to satisfy the very strong constraints from equivalence principle tests [4]. The second motivation for ultralight scalars (or pseudoscalars) are the pseudo-Nambu Goldstone bosons of spontaneously broken global symmetries that have an additional breaking due to nonperturbative effects or to a weak explicit breaking [27]-[30]. In simple examples the scalar potential of such PNGB's respects a discrete periodic remnant of their original shift symmetry. We will assume that scalar–tensor gravity implementations of this idea imply periodic functions of θ coupling to R.

2. Scalar–tensor theories

Scalar–tensor theories are most simply defined as conventional general relativity with a real scalar field coupled directly to the Ricci curvature. Viable models of this type must have weak couplings between the scalar and conventional matter or radiation. In the approximation where the scalar is decoupled from matter, a general model can be defined in the Jordan frame:

$$S = S_{\rm grav}(g_{\mu\nu},\theta) + S_{\rm scalar}(g_{\mu\nu},\theta) + S_{\rm matter}(g_{\mu\nu},\psi_{\rm matter}) \; ; \tag{2.1}$$

$$S_{\rm grav} = -\frac{k^2}{4}\int d^4x\sqrt{-g}\; D(\theta)R \; ; \tag{2.2}$$

$$S_{\rm scalar} = \int d^4x\sqrt{-g}\;\left[\frac{1}{2}Z(\theta)g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\theta)\right] \; . \tag{2.3}$$

$$\tag{2.4}$$

where $k^2 = 1/4\pi G$, $\theta(x)$ is the scalar field rescaled by k to be dimensionless, $D(\theta)$, $Z(\theta)$ and $V(\theta)$ are arbitrary functions, and $\psi_{\rm matter}$ denotes generic matter and radiation. We use the metric signature (+1,−1,−1,−1) and Wald's convention for the sign of the Riemann curvature [31].

2.1 frames

At the classical level, one is free to perform arbitrary rescalings of the metric field and the scalar, thus obtaining many other frames that are classically equivalent to the Jordan frame [32]-[34]. For example, θ can be redefined so as to make $Z(\theta)/k^2 = 1$, giving a

conventional kinetic term. However this frame is problematic if the original $Z(\theta)$ has zeros; note that $Z \to 0$ is an indication that the scalar is becoming either strongly coupled or nondynamical.

A conformal transformation of the metric can always be found such that in the new frame $D(\theta) = 1$. This transformation to the Einstein frame recovers the conventional Einstein-Hilbert action, but introduces a direct coupling between the scalar field and matter in $S_{\rm matter}$. This transformation is also problematic if $D(\theta)$ has zeros, an indication that gravity is becoming strongly coupled.

By a combination of a conformal transformation and a redefinition of the scalar, it is also possible to find a frame where $Z(\theta) = 0$. Since the scalar is then nondynamical, it can be eliminated by solving the constraint provided by its equation of motion. Thus in this frame the scalar–tensor theory becomes an $f(R)$ theory of modified general relativity.

For most purposes the physics of scalar–tensor theories is more transparent in the Jordan frame. Avoiding other frames also avoids the difficult question of the status of these classical field redefinitions in the full quantum theory.

2.2 equations of motion

In the absence of matter, the equations of motion for the general scalar–tensor theory with a Friedmann-Robertson-Walker metric ansatz become:

$$H^2 = \frac{Z}{3k^2D}\dot{\theta}^2 - \frac{\dot{D}}{D}H + \frac{2}{3k^2D}V \; ; \tag{2.5}$$

$$\dot{H} = \frac{\dot{D}}{2D}H - \frac{\ddot{D}}{2D} - \frac{Z}{k^2D}\dot{\theta}^2 \; ; \tag{2.6}$$

$$0 = \ddot{\theta} + 3H\dot{\theta} + \frac{1}{2}\frac{Z'}{Z}\dot{\theta}^2 + \frac{k^2}{4}\frac{D'}{Z}R + \frac{V'}{Z} \; , \tag{2.7}$$

where a prime indicates variation with respect to the scalar field θ. Here $H = \dot{a}/a$ is the Hubble expansion rate, and the Ricci curvature is given by

$$R = -6(\dot{H} + 2H^2) \; . \tag{2.8}$$

We have assumed that the spatial curvature in the FRW ansatz vanishes.

The third equation of motion (2.7) is redundant to the first two, which together form a coupled set of nonlinear differential equations for $H(t)$ and $\theta(t)$. The first equation of motion is the Friedmann equation for scalar-tensor cosmology in the absence of matter and radiation. Combining it with the second equation of motion, one can derive a conventional continuity equation:

$$\dot{\rho_{\rm eff}} + 3H(\rho_{\rm eff} + p_{\rm eff}) = 0 \; , \tag{2.9}$$

where the conserved energy density $\rho_{\rm eff}$ and the corresponding pressure $p_{\rm eff}$ are given by:

$$\rho_{\rm eff} = \frac{3}{2}k^2H^2 \; ; \tag{2.10}$$

$$p_{\rm eff} = -\frac{1}{2}Z\dot{\theta}^2 - V - k^2\dot{H} + \frac{3}{2}k^2\left[H\dot{D} + H^2(D-1)\right] \; . \tag{2.11}$$

Of course the first relation (2.10) is a just a rewriting of the Friedmann equation in its conventional form $H^2 = 2\rho/3k^2$. It is important to keep in mind that ρ_{eff} and p_{eff} differ from the flat space energy density and pressure:

$$\rho_{\text{eff}} = \rho + \Delta\rho \; ; \tag{2.12}$$

$$p_{\text{eff}} = = p + \Delta p \; , \tag{2.13}$$

where

$$\rho = \frac{1}{2} Z\dot{\theta}^2 + V \; ; \tag{2.14}$$

$$p = \frac{1}{2} Z\dot{\theta}^2 - V \; ; \tag{2.15}$$

$$\Delta\rho = \frac{3}{2} k^2 H^2 - \frac{1}{2} Z\dot{\theta}^2 - V \; ; \tag{2.16}$$

$$\Delta p = \frac{k^2}{2} \left[\ddot{D} + 2H\dot{D} + (2\dot{H} + 3H^2)(D - 1) \right] \; . \tag{2.17}$$

The effective equation of state for the scalar is given by

$$p_{\text{eff}} = w_{\text{eff}} \rho_{\text{eff}} \; ; \tag{2.18}$$

$$w_{\text{eff}} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2} \; . \tag{2.19}$$

Assuming a quintessence role for the scalar θ, the effective parameter w_{eff} would be what is extracted, *e.g.*, from Type Ia supernovae observations. There is no simple relation between w_{eff} and $w = p/\rho$.

2.3 tracking solutions for scalar-tensor cosmologies

The FRW solutions that are of greatest cosmological interest are those for which the time evolution of the scalar field θ tracks the expansion rate. The simplest ansatz for this kind of behavior is

$$\dot{\theta} = -bH \; , \tag{2.20}$$

where b is a constant. We will restrict ourselves to solutions of this type, although more complicated scenarios are certainly possible.

Imposing the ansatz (2.20), the equations of motion (2.5-2.6) become over-constrained. Thus solutions of the desired type are only obtained if the input functions $D(\theta)$, $Z(\theta)$, and $V(\theta)$ obey a constraint, given by

$$Z = \frac{3k^2}{b^2} (D - bD' - v) \; , \tag{2.21}$$

where we have introduced the dimensionless notation:

$$V \equiv \frac{3}{2} v k^2 H^2 \; . \tag{2.22}$$

The equations of motion can then be solved for the effective scalar equation of state parameter $w_{\rm eff}$:

$$w_{\rm eff} = 1 - \frac{2}{3}\,\frac{6v + 2bD' - b^2 D''}{2D - bD'} \ , \tag{2.23}$$

in terms of which the Hubble rate is given by:

$$H = H_0 \exp \frac{1}{b} \int^{\theta} d\theta' \left[1 + w(\theta') \right] \ . \tag{2.24}$$

Note that, taking Z, D and v as functionals of θ/b, a rescaling of b can be undone by a rescaling of θ (which also implies an overall rescaling of the kinetic function Z). Thus we can take $b = 1$ from now on with no loss of generality, writing

$$z = Z/k^2 = 3(D - D') - 2v \ , \tag{2.25}$$

and

$$w_{\rm eff} = 1 - \frac{2}{3}\,\frac{6v + 2D' - D''}{2D - D'} \ . \tag{2.26}$$

There is no known reason why the coupling functionals $Z(\theta)$, $D(\theta)$ and $V(\theta)$ should obey the constraint (2.21) exactly. However it is certainly plausible that they satisfy this relation approximately during a certain cosmological epoch.

2.4 phantoms, ghosts and strong coupling

Generic scalar–tensor lagrangians will lead to behaviors that are unphysical, unstable or singular at the classical level, the quantum level, or both. A partial list of possibilities includes

- If $Z(\theta) < 0$ during any epoch, the solution has a kinetic ghost. In the cosmological context kinetic ghosts are known as phantoms [35]-[38]. They violate the weak energy condition $p+\rho \geq 0$ and the dominant energy condition $\rho \geq |p|$. Phantoms generically lead to singularities, dangerous instabilities, and pathologies in the ultraviolet behavior of the underlying field theory [39, 40]. A successful scalar–tensor model with a phantom epoch would have to address all of these difficulties.

- If $D(\theta) < 0$ during any epoch, then the graviton becomes a kinetic ghost. Theories of this type are believed to be unphysical [12].

Our philosophy will be to avoid such behaviors. We want to understand the cosmological significance of scalar-tensor theories *per se*, not as examples of other exotica.

Time variation of the vacuum expectation value of D corresponds to a variation in the effective strength of the gravitational coupling. The magnitude of $\dot{D}$ is subject to strong observational bounds during certain epochs, especially the present day [41]-[45]. D approaching zero corresponds to gravity becoming strong. Time variation of the vacuum expectation value of Z, after a rescaling, corresponds to changing the self-coupling of the

scalar field θ, as well as its couplings to matter. The magnitude of these latter couplings are subject to strong observational upper bounds, at least during the present epoch. Z approaching zero corresponds to the scalar sector becoming strongly coupled, a possibility reminiscent of the discussions in [46].

When we exhibit solutions that have $D \to 0$ and/or $Z \to 0$ at some time in the past, we will consider these as a breakdown of the modelling of the physics due to scalar-tensor gravity sector becoming strongly coupled.

3. De Sitter expansion with a negative cosmological constant

Even without resorting to phantoms or other exotica, the scalar-tensor equations of motion have many remarkable solutions. One dramatic example is obtained by asking for a de Sitter solution, *i.e.* $H =$ constant and $w_{\rm eff} = -1$. We will also suppose that the scalar potential consists entirely of a cosmological constant: $v(\theta) = v_0$. Inserting these ansatze into (2.26) yields a constraint on the coupling $D(\theta)$:

$$D'' - 5D' + 6(D - v_0) = 0 \; . \tag{3.1}$$

The solution of this constraint with the additional properties $D(0) = 1$, $D'(0) = 0$ is given by:

$$D(\theta) = v_0 + 3(1 - v_0)\mathrm{e}^{2\theta} - 2(1 - v_0)\mathrm{e}^{3\theta} \; ; \tag{3.2}$$

$$z(\theta) = 3(1 - v_0)\left(-3\mathrm{e}^{2\theta} + 4\mathrm{e}^{3\theta}\right) \; , \tag{3.3}$$

where we have also assumed a form for the kinetic function satisfying the constraint (2.25).

It is easily verified that the scalar–tensor theory defined by (3.2) gives an exactly de Sitter solution $H = H_0$, $w_{\rm eff} = -1$, for any value of the cosmological constant v_0, including a negative cosmological constant. Furthermore, provided that $v_0 < 1$, there is an epoch which includes the present time ($\theta = 0$) where both $D(\theta)$ and $Z(\theta)$ are positive. Thus the cosmology that we are describing does not rely on exotic matter or ghosts. It does carry the price that the description breaks down at some time both in the past and in the future, where strong coupling occurs in the scalar–tensor sector.

The solutions are even more interesting if we add conventional matter sources. Since the scale factor a is proportional to $\exp(-\theta)$, conventional matter will appear on the right hand side of the Friedmann equation as a rescaled energy density

$$\rho_m = \Omega_m \mathrm{e}^{3\theta} \; . \tag{3.4}$$

The exact solution to the equations of motion is

$$\frac{H^2}{H_0^2} = 1 + \frac{\Omega_m}{(1 - v_0)} \log\left[v_0 + (1 - v_0)\mathrm{e}^{3\theta}\right] \; . \tag{3.5}$$

The resulting behavior is shown in Figure 1, for the particular case of $\Omega_m = 0.25$ and negative cosmological constant $v_0 = -1$. Positive θ corresponds to the past, negative θ to the future. In the domain plotted both $D(\theta)$ and $Z(\theta)$ are strictly positive.

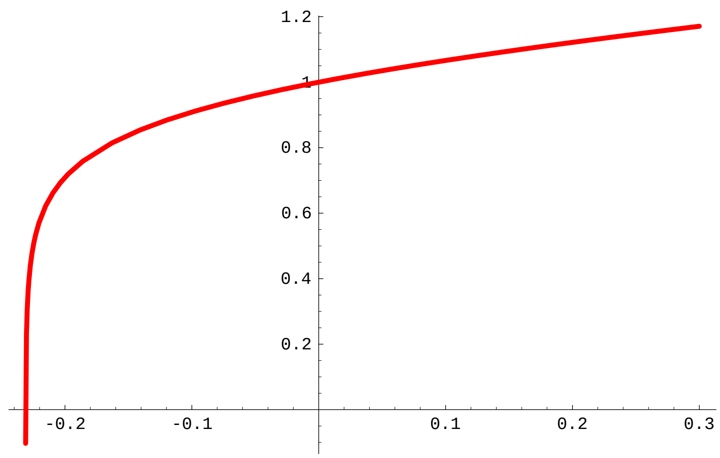

Figure 1: The squared Hubble rate as a function of the scalar field expectation value θ. H is normalized such that $H = H_0$ when $\theta = 0$. The scalar–tensor theory is defined with a negative cosmological constant $v_0 = -1$ (*i.e.* a constant scalar potential $V = -\frac{3}{2}k^2H_0^2$). Matter has been added corresponding to $\Omega_m = 0.25$.

For positive θ, the Hubble rate is nearly constant, *i.e.* de Sitter-like, and slightly decreasing due to the matter source. However in the future the negative cosmological constant begins to dominate, and H^2 goes rapidly to zero, transitioning from a nearly de Sitter metric to an anti-de Sitter metric. Of course, a tiny negative cosmological constant will always assert itself at some point in the future when other sources have diluted. What is remarkable here is that the negative cosmological constant is of the same magnitude as the chimeric positive vacuum energy mocked up by the effects of scalar–tensor gravity!

As expected, in the case where v_0 is positive and less than one, the expansion is de Sitter-like at $\theta = 0$ and becomes increasing de Sitter-like in the future. In this case the effects of scalar–tensor gravity and real positive vacuum energy conspire together. In the special case $v_0 = 1$, the solution reduces back to ordinary inflating general relativity.

The coupling functions D and z shown in (3.2) have the form of linear combinations of exponentials of θ. These are reminiscent of the effective low energy action of string theory, with θ representing the dilaton or other related moduli fields, and higher powers of $\exp(\theta)$ representing higher orders in the string coupling.

Damour and Polyakov examined long ago [47] the possibility of a string dilaton or similar modulus surviving as a massless field in a phenomenolgically realistic string compactification. They pointed out that very stringent observational constraints on this scenario can be satisified via a dynamical attraction to a local maximum of $D(\theta)$. This is precisely what occurs in our example, where D has a local maximum at $\theta = 0$.

In order to avoid the strongest observational bounds on the time variation of G, θ would have to be very close to this local maximum today. For example the constraint from the Cassini spacecraft [41] requires that $\theta \leq 0.002$ today, for the model with a negative cosmological constant $v_0 = -1$. Even with such a tuning the model is problematical as an explanation of the present day accelerated expansion, since already at a redshift of 0.1 $\dot{G}/G$ is twice as large as it is now.

4. PNGB gravity

In the previous example the form of the coupling functions was inspired by the dilaton and other moduli, the massless real scalar fields of string theory. The other well-motivated approach to very light scalars or pseudoscalars are pseudo-Nambu Goldstone bosons of a spontaneously broken $U(1)$ symmetry. The PNGBs have a shift symmetry which prevents them from appearing in the coupling functions D and Z or from having a nontrivial potential V. However nonperturbative effects or a weak explicit breaking can change this picture. The simplest assumption is that D, Z and V are restricted to be periodic functions of θ, preserving a discrete remnant $\theta \to \theta + 2\pi$ of the original shift symmetry.

An interesting example of a scalar-tensor model of this type is defined by

$$D(\theta) = 1 + \lambda(\cos\theta - 1) \; ; \tag{4.1}$$

$$Z(\theta) = \frac{1}{2}\lambda k^2(\cos\theta + \sin\theta) \; ; \tag{4.2}$$

$$V = \frac{3}{2}k^2 H_0^2 \left[1 + \lambda\left(\frac{5}{6}(\cos\theta + \sin\theta) - 1\right)\right] \; . \tag{4.3}$$

When the dimensionless parameter λ is taken to be small, $\dot{G}/G$ effects are suppressed. However since the kinetic coupling Z is proportional to λ, taking λ small also increases the strength of λ self-couplings as well as any couplings of the PNGB θ to ordinary matter. This trade-off between suppressing variations of G and suppressing couplings to matter is a general feature of scalar–tensor models.

In this model the effective equation of state parameter w_{eff} is exactly -1, giving a de Sitter solution. For small λ this solution is obviously a perturbation of the standard de Sitter solution arising from a positive cosmological constant. The coupling $D(\theta)$ is positive for all values of θ. The kinetic coupling $Z(\theta)$ vanishes at $\theta \simeq 2.35$, corresponding to a redshift of about 10, indicating that the PNGB sector becomes strongly coupled.

Taking $\theta = 0$ to represent the present day, we note that $D(0) = 1$ and that D is at a local maximum. We can take a reasonably small value of λ, $\lambda = 0.05$, and investigate the constraints on the model. All of the present day bounds $\dot{G}/G$ and post-Newtonian parameters are satisfied, provided we have tuned the present day to coincide with $\theta = 0$ within about 3 per cent accuracy. Furthermore in this model $\dot{G}/G$ oscillates, so at no time in the past did G differ from it's current value by more than 10 percent.

5. General case

As noted in the introduction, a remarkable feature of scalar–tensor gravity is that it allows one to obtain *any* equation of state starting from *any* scalar potential, via a suitable choice of the coupling functions D and Z. The major caveat is that the required D and Z may not be positive, so only a subset of such models are manifestly physical.

By the same token, one can obtain any equation of state starting from any choice of $D(\theta)$. As an example, suppose that we want to reproduce the oscillatory equation of state of the Slinky quintessence model [48]-[51]:

$$w_{\text{eff}} = -\cos 2\theta \; . \tag{5.1}$$

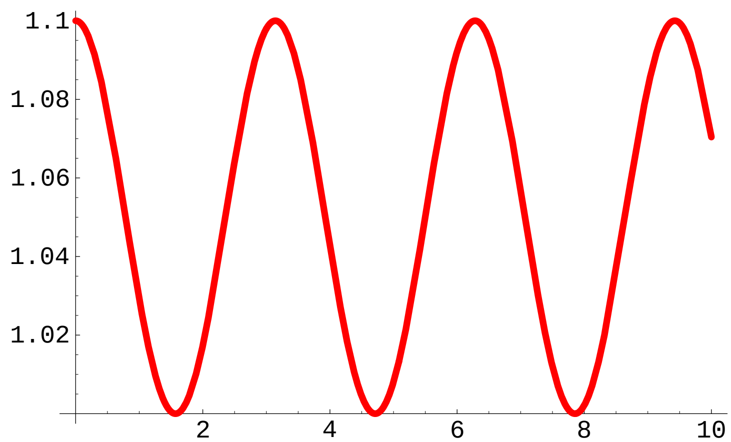

Figure 2: The gravitational coupling functional $D(\theta)$ as a function of θ.

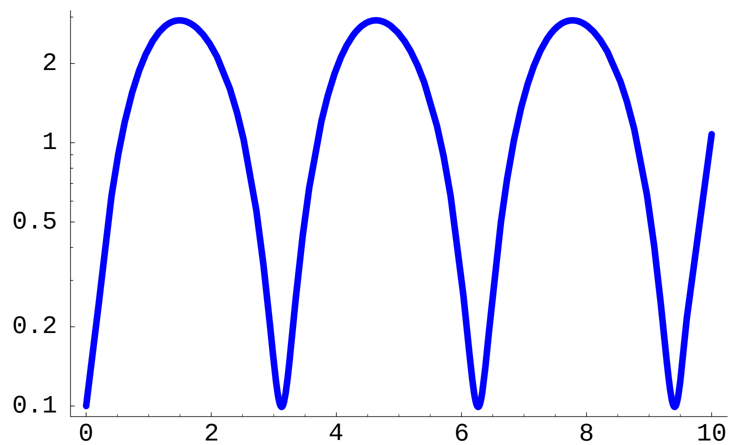

Figure 3: The kinetic functional $z(\theta)$ as a function of θ.

At the same time, we will assume a simple oscillatory term in the coupling D:

$$D(\theta) = 1 + \lambda \cos^2\theta \; , \tag{5.2}$$

where we have in mind that λ is a small parameter. A scalar–tensor model with the desired equation of state is then obtained by substituting (5.2) into (2.21) after first substituting (5.2) and (5.1) into the following expression for $v(\theta)$:

$$v(\theta) = \frac{1}{6}D'' - \frac{5}{6}D' + D + \frac{1}{4}(1 + w_{\rm eff})(D' - 2D) \; . \tag{5.3}$$

Taking $\lambda = 0.1$, we obtain a scalar–tensor version of the Slinky model for which $D(\theta)$ and $Z(\theta)$ are oscillatory and strictly positive for all values of θ, as shown in Figures 2 and 3. The model satisfies all the present day constraints on $\dot{G}/G$ and the post-Newtonian parameters, provided that θ is tuned to be within about 2 per cent of a local minimum or maximum. Because $\dot{G}/G$ is oscillatory, the magnitude of G never varies by more than 9 per cent from its current value.

6. Conclusion

We have seen that scalar-tensor gravity models can have remarkably simple self-accelerating solutions, without resorting to phantoms or other ghosts. These solutions are made possible by the fact that the conserved energy associated to the scalar is not the conventional energy that one would read off from the lagrangian in the flat space limit.

As we have seen, this self-accelerating feature can even overcome, for a time, the effects of a negative cosmological constant of similar magnitude. In such a scenario, our immediate cosmological future exactly contradicts what one would predict from a naive extrapolation of the current expansion. Although the model we exhibited was not entirely realistic, it has an intriguing connection to previous attempts to construct realistic string models with ultralight moduli.

Scalar-tensor models are very constrained by observational data. It does not appear likely that one could account for dark energy entirely from the effects of scalar-tensor gravity, especially in a framework where the gravity sector is manifestly weakly coupled and ghost–free.

However we gave two examples of realistic models where the novel properties of scalar-tensor gravity play an important role in the current accelerated expansion. These models are somewhat tuned in order to satisfy present day limits on the time variation of G. They predict interesting variations of G, on the order or 10% , at earlier times. Improved CMB [45] or neutron star [44] constraints would directly test such scenarios, as would galaxy cluster based tests of modified gravity [52, 53].

More generally, our examples show the importance of using a broad-based observational approach to dark energy. The ambitious Stage IV dark energy probes currently planned [54] are certainly not sufficient by themselves [55, 56] to disentangle scalar-tensor effects from quintessence and other scenarios.

The self-accelerating properties of scalar-tensor models look promising for models of primordial inflation. It appears that such models could have virtues similar to models of hybrid inflation [57, 58]. Variations of G at times prior to Big Bang Nucleosynthesis are hardly constrained. In this arena it also seems more promising to forge a concrete link between scalar-tensor gravity and string theory.

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